Properties of Boolean Formulas

Commutativity:

$$x_1 \wedge x_2 = x_2 \wedge x_1, \qquad x_1 \vee x_2 = x_2 \vee x_1, \qquad x_1 \oplus x_2 = x_2 \oplus x_1, \qquad x_1 \leftrightarrow x_2 = x_2 \leftrightarrow x_1.$$

Associativity:

- $x_1 \wedge (x_2 \wedge x_3) = (x_1 \wedge x_2) \wedge x_3$
- $x_1 \vee (x_2 \vee x_3) = (x_1 \vee x_2) \vee x_3$
- $x_1 \oplus (x_2 \oplus x_3) = (x_1 \oplus x_2) \oplus x_3$
- $x_1 \leftrightarrow (x_2 \leftrightarrow x_3) = (x_1 \leftrightarrow x_2) \leftrightarrow x_3$

Distributivity:

- $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$
- $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$
- $A \vee (B \to C) = (A \vee B) \to (A \vee C)$
- $A \to (B \wedge C) = (A \to B) \wedge (A \to C)$
- $A \to (B \vee C) = (A \to B) \vee (A \to C)$
- $A \to (B \to C) = (A \to B) \to (A \to C)$

Other usefull properties:

- $A \vee (A \wedge B) = A, \quad A \wedge (A \vee B) = A$
- $A \wedge A = A, \quad A \vee A = A$
- $A \wedge \neg A = 0, \quad A \vee \neg A = 1$
- $A \wedge 0 = 0, \quad A \vee 0 = A$
- $A \wedge 1 = A, \quad A \vee 1 = 1$
- $A \to B = \neg A \vee B, \quad A \vee B = \neg A \to B$