Discrete Mathematics **DSBA**

Seminar 1. Basics of logic

1. Find the word for which the following proposition is **false**

"The first letter is vowel $\to$ (The second letter is vowel $\vee$ The last letter is consonant)"

1) Heat 2) Owl 3) Amphora 4) Parade

2. Prove the following statements **a)** $x \to y = \bar{x} \vee y$; **b)** $\overline{x \wedge y} = \bar{x} \vee \bar{y}$ **c)** $\overline{x \to y} = x \wedge \bar{y}$.

3. A Boolean function f is defined by the vector of values: $f(x_1, x_2, x_3) = 10100101$.

Describe f via **a)** Truth table

b) Disjunctive Normal Form (DNF)

c) Conjunctive Normal Form (CNF)

Which variables of f are **d)** significant? **e)** dummy?

4. Let $f_{01}(x_1, x_2, \dots, x_n)$ be a Boolean function such that $f_{01}(x_1, \dots, x_n) = 0$ either on the input $(0, 0, \dots, 0)$ or on the input $(1, 1, \dots, 1)$. Describe f_{01} in CNF.

5. Construct DNF expansions for the Boolean functions described by formulas:

a) $x_1 \oplus x_2 \oplus (x_1 \wedge x_2)$; **b)** $(x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (x_1 \vee x_4) \wedge \dots \wedge (x_1 \vee x_9)$;

c) * $\bigwedge\limits_{1 \leqslant i < j < k \leqslant 5} (x_i \vee x_j \vee x_k) \wedge (\bar{x}_i \vee \bar{x}_j \vee \bar{x}_k)$.

6. Prove the following expansion formulas (Shannon expansions):

a) $f(x_1, x_2, \dots, x_n) = (\bar{x}_1 \wedge f(0, x_2, \dots, x_n)) \vee (x_1 \wedge f(1, x_2, \dots, x_n))$;

b) $f(x_1, x_2, \dots, x_n) = ((1 \oplus x_1) \wedge f(0, x_2, \dots, x_n)) \oplus (x_1 \wedge f(1, x_2, \dots, x_n))$.

7. A Boolean function $\mathrm{MAJ}(x_1, x_2, \dots, x_n)$ is false when $n/2$ or more arguments are false, and true otherwise. (Equals to the majority value of the input variables). Prove that MAJ has a DNF-expansion such that each literal is a variable (there is no negation in each clause).

8. Prove that there is no boolean function $f(x, y)$ with both significant variables such that

$$\overline{f(x, y)} = f(\bar{x}, \bar{y}).$$

Discrete Mathematics **BSBA**

Home assignment 1

1. a, b, c — are integers such that the following proposition holds

$$\neg(a = b) \wedge ((b < a) \to (2c > a)) \wedge ((a < b) \to (a > 2c))$$

Find a if $c = 7, b = 16$.

2. Prove the formula

$$1 \oplus x_1 \oplus x_2 = (x_1 \to x_2) \wedge (x_2 \to x_1).$$

3. Find significant and dummy variables of the following functions:

a) $f(x_1, x_2, x_3) = 00111100$; **b)** $g(x_1, x_2, x_3) = (x_1 \to (x_1 \vee x_2)) \to x_3$.

4. Construct for the function f from the previous problem **a)** a DNF expansion. **b)** a CNF expansion.

5. Prove the expansion formula

$$f(x_1, \ldots, x_n) = (x_1 \vee f(0, x_2, \ldots, x_n)) \wedge (\bar{x}_1 \vee f(1, x_2, \ldots, x_n)).$$

6. Construct a DNF expansion for the function $x_1 \to (x_3 \wedge \neg x_2 \leftrightarrow x_1)$.

7. Construct a DNF expansion for the function $(x_1 \to x_2) \wedge (x_2 \to x_3) \wedge \ldots \wedge (x_7 \to x_8)$.

8. A Boolean function $\mathrm{PAR}(x_1, x_2, \ldots, x_n)$ (parity) equals 1 if the number of ones among the values of variables $x_1, x_2, \ldots, x_n$ is odd an equals zero if it is even.

a) Construct a formula for $\mathrm{PAR}(x_1, x_2, \ldots, x_n)$. It is allowed to use logical connectives $\wedge, \vee, \neg, \oplus, \to$.

b) Is it possible to expand $\mathrm{PAR}(x_1, x_2, \ldots, x_n)$ via DNF without negations?