Discrete Mathematics **DSBA**

Seminar 2. Set Theory and Logic

1. Prove that for any sets A, B and C the following equalities hold:

a) $A \setminus (A \setminus B) = A \cap B$;

b) $B \cup (A \setminus B) = A \cup B$;

c) $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$;

d) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$.

Use both Euler-Venn diagrams and corresponding propositional logic equations.

2. Prove that the following inclusion holds

$$(A_1 \cap A_2 \cap \cdots \cap A_n) \triangle (B_1 \cap B_2 \cap \cdots \cap B_n) \subseteq (A_1 \triangle B_1) \cup (A_2 \triangle B_2) \cup \cdots \cup (A_n \triangle B_n)$$

for arbitrary sets A_i, B_i.

3. Prove that the following equality holds

$$(A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n) \setminus (B_1 \cup B_2 \cup \cdots \cup B_n) = (A_1 \setminus B_1) \cap (A_2 \setminus B_2) \cap \cdots \cap (A_n \setminus B_n)$$

for arbitrary sets A_i, B_i.

4. Express the characteristic function $\chi_{A \triangle B}(x)$ via an equation that contains only (but not necessary all) $\chi_A(x), \chi_B(x)$ and

a) boolean operations $\wedge, \vee, \neg$;

b) arithmetic operations $+, -, \times$.

5. Express $|A \triangle B|$ via an equation that contains only $\chi_A(x), \chi_B(x)$ and arithmetic operations.

6. Consider statements of the form $X = Y$ where X and Y contain only set variables and operations $\cap$, $\cup$, $\setminus$. Prove that if such a statement is false then there is a counterexample such that each set in it either an empty set or contains the only element, say 1.

Discrete Mathematics **BSBA**

Home Assignment 2

1. Does the following equality hold for arbitrary sets A and B?

$$(A \setminus B) \cap ((A \cup B) \setminus (A \cap B)) = A \setminus B$$

2. Does the following equality hold for arbitrary sets A, B and C?

$$((A \setminus B) \cup (A \setminus C)) \cap (A \setminus (B \cap C)) = A \setminus (B \cup C)$$

3. Does the following equality hold for arbitrary sets A, B and C?

$$(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$$

4. Does the following inclusion hold for arbitrary sets A and B?

$$(A \cup B) \setminus (A \setminus B) \subseteq B$$

5. Sets A, B, X, Y satisfy the properties $A \cap X = B \cap X$, $A \cup Y = B \cup Y$. Does the equality $A \cup (Y \setminus X) = B \cup (Y \setminus X)$ necessarily hold?

6. Fix a decreasing sequence of sets $A_1 \supseteq A_2 \supseteq A_3 \supseteq \cdots \supseteq A_n \supseteq \ldots$ such that $A_1 \setminus A_4 = A_6 \setminus A_9$. Prove that $A_2 \setminus A_7 = A_3 \setminus A_8$.

7. Let A, B, C, D be closed line segments (sets of the form $[x_1, x_2], x_i \in \mathbb{R}$) such that $A \triangle B = C \triangle D$. Does it imply the inclusion $A \cap B \subseteq C$?