

Seminar 4. Combinatorics: Rule of Product

1. There are 3 carnations, 4 roses and 5 tulips in a flower shop. Count the number of bouquets that could be composed according to the following rules (there is at least one flower in a bouquet).
 - a) All the flowers of a bouquet are of the only species.
 - b) There are odd number of flowers of each species.
 - c) No restrictions.
2. Count the number of 9-digit numbers the digits of which are arranged in the descending order (each next digit is less than the previous one).
3. Find the probability that in a random 4-letter word in the English alphabet, there is at least one vowel? (A total number of letters is 26, 5 of them are vowels: surprisingly “Y” is not a vowel!)

Comments: *By probability we mean the ratio of the number of favorable to the number of possible outcomes.*

4. Find the number of Boolean functions of n variables with (exactly) k unit points.
5. A staircase consists of 13 steps, not counting the upper and lower platforms. Going down, you can jump over some steps (even over all). How many ways can you go down the stairs?
6. Find the number of Boolean functions of n variables that take the value of 1 only on those inputs that contain exactly k ones (but not necessarily on all such sets). That is $f(\alpha_1, \dots, \alpha_n) = 1$ only if there are exactly k alphas equal to 1.
7. 10 people have randomly lined up. Find the probability that **a)** Alice, Bob and Clare stand in a row (in no particular order); **b)** Alice is ahead of Bob; **c)** Alice and Bob are not standing next to each other.
8. How many ways are there to make 6 pairs of 12 people?
9. How many ways are there to arrange 12 white and 12 black checkers on the black squares of a chessboard?
10. There are three kinds of donuts: with chocolate, with cinnamon and with nut. Donuts are packed in standard boxes of 18 items in each. Each box can contain donuts of all kinds. The order of the donuts in the box is not significant. How many different boxes of donuts can one make, provided that in a box there are at most 9 chocolate donuts, at most 3 cinnamon donuts, and at most 9 donuts with nuts?
11. How many ways are there to write in a row the numbers from 0 to 9 such that the even numbers go in the ascending order, and the odd ones go in the descending order?

Home Assignment 4

1. Find the number of Boolean functions f of n variables such that $f(0, 0 \dots, 0) = 0$ and $f(1, 1 \dots, 1) = 1$.
2. A Boolean function is called symmetric if its value depends only on the number of ones on the input, but not on their order. Count the number of symmetric Boolean functions of n variable
3. There are 6 candidates for 6 vacancies. How many ways are there to hire candidates? (Each vacancy must be filled.)
4. **a)** What kind of numbers are more than the others among the first million (from 1 to 1 000 000): that contain at least one digit 1 in the decimal representation or that doesn't?
b) The same question is for the first 10 million numbers.
5. Find the probability that in the decimal representation of a random six-digit number, there will be at least two identical digits.
6. Four cards have been randomly drawn from a 36-card deck. What is the probability that two of them are red and two are black?
7. Find the number of 7-digit numbers with exactly two even digits such that there is an odd digit in front of each (on the preceding position).
8. In how many ways can one accommodate 7 students in three rooms: single, double and quadruple?