Discrete Mathematics **DSBA**

Seminar 5. Binomial coefficients

1. Chess rook stands in the left-most squared of a non-standart chessboard of size 1 $times 30$. It can go only to the right at any (positive) number of squares per move.

a) In how many ways can the rock reach the right-most square?

b) In how many ways can the rock reach the right-most square by exactly 7 moves?

2. Find the coefficient

a) of a term x^3y^7 in the expansion of $(2x-y)^{10}$;

b) of a term $x_1^3x_2x_4^5x_5$ in the expansion of $(x_1+x_2+x_3+x_4+x_5)^{10}$.

3. Prove that

a) $\displaystyle\sum_{j=0}^{k}\binom{r}{j}\binom{s}{k-j}=\binom{r+s}{k}$;

b) $\displaystyle\sum_{j=0}^{n}\binom{j}{k}=\binom{n+1}{k+1}$.

(Try to find out a combinatorial proof.)

4. How many solutions of an equation $x_1+x_2+x_3+x_4+x_5=11$ in non-negative integers ($\mathbb{N}$) are there?

5. In how many ways you can give 15 identical candies to 7 children so that each child gets at least one candy?

6. In how many ways you can choose 6 numbers from 1 to 15, so that among them there are not two, differing by one?

7. An *expansion* of a positive integer n is a sequence of positive integers $x_1, x_2, \ldots, x_k$ such that $x_1+x_2+\cdots+x_k=n$. Find the number of expansions of n in odd terms.

8. Prove that

$$\binom{n+k+1}{k}=\binom{n}{0}+\binom{n+1}{1}+\binom{n+2}{2}+\ldots+\binom{n+k}{k};$$

Discrete Mathematics **DSBA**

Home Assignment 5

1. A robot moves in a coordinate plane. At each step, it can increase one coordinate by 1 or both coordinates by 2. How many ways are there to move a robot from the point $(0,0)$ to point $(4,5)$?

2. Which term of $(1+2)^n$ in the binomial expansion is the largest one?

3. Find the number of words of length n over the alphabet $\{0,1\}$ in which there are no two ones in a row.

4. Prove the formula $\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$ via combinatorial proof.

5. Which number is greater: $\binom{F_{1000}}{F_{998}+1}$ or $\binom{F_{1000}}{F_{999}+1}$? Here F_n stands for n-th Fibonacci number.

6. Prove the formula (try to find a combinatorial proof!)

$$\sum_{0\leqslant k\leqslant (n+1)/2} \binom{n-k+1}{k} = F_{n+2}.$$

The following problems are moved to the next home assignments.

7. How many ways are there to place 20 of various books on 5 shelves if each shelf can hold all 20 books? Arrangements that differ in the order of books on the shelves are different.

8. A student council of 8 people chooses a chairman from among its members by secret voting. Everyone can cast one vote for any student council member. The result of a vote is the number of votes cast for each candidate. How many different voting results are there?