Discrete Mathematics **DSBA**

Seminar 6. Combinatorics and functions

1. How many solutions of an equation $x_1 + x_2 + x_3 + x_4 + x_5 = 11$ in non-negative integers ($\mathbb{N}$) are there?

2. In how many ways you can give 15 identical candies to 7 children so that each child gets at least one candy?

3. In how many ways you can choose 6 numbers from 1 to 15, so that among them there are no two, differing by one?

4. Count the number of **a)** totally defined functions **b)** (partially defined) functions **c)** injections from a 5-element set to a 9 element set.

5. A function f is defined from a set $\{1, 2, \ldots, 8\}$ to a set $\{a, b, \ldots, e\}$ as follows:

$$f : 1 \mapsto a, \quad 2 \mapsto a, \quad 3 \mapsto c, \quad 4 \mapsto d, \quad 5 \mapsto c, \quad 7 \mapsto d.$$

Find **a)** $\mathrm{Dom}(f)$ **b)** $\mathrm{Range}(f)$ **c)** $f(\{1, 2, 3\})$ **d)** $f^{-1}(c)$ **e)** $f(\{1, 2, 3, 5, 6\})$ **f)** $f^{-1}(\{a, b, c\})$

6. A (partial) function g from the positive integers to themselves returns the greatest prime devisor of a number on the input.

a) Find $\mathrm{Dom}(g)$.

b) Find $\mathrm{Range}(g)$.

c) Is it true that if X is a finite set then $g^{-1}(X)$ is also a finite set?

7. In how many ways can one distribute k (different) gifts to n children such that

a) each child gets exactly one gift

b) the first child gets k_1 gifts, the second gets k_2 gifts, …, the n-th gets k_n gifts ($k_1 + k_2 + \cdots + k_n = k$)

Name the type of function corresponding to such a distribution and describe the function itself: name the domain and the range.

8. Find the number of **a)** non-increasing injections $f\colon \{1, 2, \ldots, n\} \to \{1, 2, \ldots, m\}$; **b)** non-increasing surjections $f\colon \{1, 2, \ldots, n\} \to \{1, 2, \ldots, m\}$.

a function is non-increasing if $x \leqslant y$ implies $f(x) \leqslant f(y)$.

9. Let f be a partial function from A to B and $X, Y \subseteq A$, $U, V \subseteq B$. Do the following statements hold for arbitrary f, A, B, X, Y, U, V?

a) $f(X \cup Y) = f(X) \cup f(Y)$;

b) The condition $f(X) = f(Y)$ implies $X \cap Y \neq \varnothing$;

c) $f^{-1}(U \cap V) = f^{-1}(U) \cap f^{-1}(V)$;

d) The condition $f^{-1}(U) = f^{-1}(V)$ implies $U = V$.

Discrete Mathematics **DSBA**

Home Assignment 5

1. How many ways are there to place 20 of various books on 5 shelves if each shelf can hold all 20 books? Arrangements that differ in the order of books on the shelves are different.

2. A student council of 8 people chooses a chairman from among its members by secret voting. Everyone can cast one vote for any student council member. The result of a vote is the number of votes cast for each candidate. How many different voting results are there?

A function h is defined from a set $\{0, 1, \dots, 8\}$ to a set $\{a, b, \dots, g\}$ for the two following problems as follows:

$$h : 1 \mapsto b, \quad 2 \mapsto c, \quad 3 \mapsto b, \quad 4 \mapsto e, \quad 5 \mapsto b, \quad 6 \mapsto e, \quad 8 \mapsto f.$$

3. Find **a)** $\mathrm{Dom}(h)$ **b)** $\mathrm{Range}(h)$ **c)** $h(\{0, 1, 2, 3, 4\})$ **d)** $h^{-1}(\{a, b, c\})$

4. Find **a)** $h^{-1}(h(\{0, 1, 2, 6, 7, 8\}))$ **b)** $h(h^{-1}(\{a, b, c, d, e\}))$.

Only answers are valid for the problems 3-4, but be ready to provide the explanations during a homework defense.

5. A function f from the set of integers to itself maps x to the smallest prime number that is greater than x^2. Prove that if the set of integers X is finite, then the preimage of this set $f^{-1}(X)$ is finite.

6. In how many ways can you distribute 9 different items to 4 people (everyone can get any number of items, even zero)? Name the type of function corresponding to such a distribution and describe the function itself: name the domain and the range.

7. Find the number of non-decreasing functions $f : \{1, 2, \dots, n\} \to \{1, 2, \dots, m\}$. The function is non-decreasing if $x \leqslant y$ implies $f(x) \leqslant f(y)$.