Discrete Mathematics **DSBA**

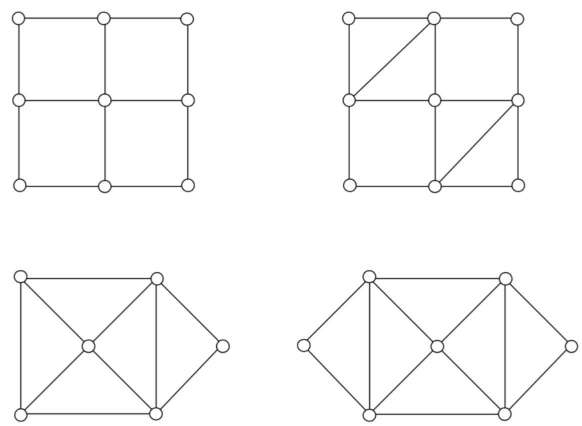
Seminar 9. Graphs III. Directed graphs

1. Vertices of a directed graph are integers from 0 to 9. Edge goes from a vertex x to a vertex y if $y - x = 3$ or $x - y = 5$. Find The number of strongly connected components in the graph.

2. Find the largest positive integer that has distinct digits and any two it's consecutive digits form a two-digit number that is divisible by 7.

3. 50 teams played in one-round volleyball tournament (each team has played with each other exactly once). We say that a team A is stronger than a team B if A won B or there is a team C such that A won against C and C won against B. Prove that the team with the most points is stronger than any other.

4. Which graphs from the picture have an Eulerian walk? Construct an Eulerian walk if it exists.



5. Find the maximum number of directed paths with given ends that can be in a directed acyclic graph on n vertices.

6. Let G be a directed graph on at least 2. It is known that there exists exactly one path from each vertex to each other vertex. Is it true that the outgoing degree of each vertex equals one ($d^{-}(u) = 1$)?

7. A sequence of numbers a_n is defined by the equation $a_{n+1} = f(a_n)$, where f is some function (defined on all numbers).

a) Show that either all elements of the sequence are different, or it is periodic, i.e. at some point (after preperiod) the numbers begin to repeat (period).

b) Show that the second case takes place if and only if $a_{2n} = a_n$ for some n.

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Home Assignment 9

1. An undirected graph G has a path that visits every edge exactly twice. Does the graph G have an Eulerian cycle?

2. Each vertex of an undirected graph G has degree 2. Prove that each (connected) component of G is a cycle.

3. G is a directed graph on n vertices. An outgoing degree (d^-) of each vertex of G equals $n-2$. How many strongly connected components can G have? Analyze all possible cases.

4. A directed graph G satisfies the following property. For any pair of vertices u and v of G there is exactly one edge: either the edge (u, v), or the edge (v, u) . Prove that G has a path that includes all vertices.

5. A professor of computer science decided to construct an order of putting clothes in the morning. For this purpose he listed the following requirements. Notation $A <_P B$ means that the professor shall put on A before B.

glasses $<_P$ *trousers* $<_P$ *belt* $<_P$ *jacket,*

trousers $<_P$ *shoes,*

glasses $<_P$ *socks* $<_P$ *shoes,*

glasses $<_P$ *watch*

Help the professor: enumerate the items so that the professor could put them on satisfying all the restrictions.

There are n cities in a magic country. Each two cities are connected by a road. Roads meet only in the cities (there are no other crossroads). An evil wizard wants to convert each road to a one-way road so that if a traveler leaves a city (by a road) than he will not be able to return to this city again.

6. Prove that **a)** the wizard can do it.

b) (after the wizard succeeded) there is a city S such that a traveler can reach each other city from S (S stands for “source”). There is a city that traveler can’t leave by a road.

7. a) Prove that there exists exactly one path that visits each city.

b) In how many ways can the wizard establish the one-way traffic to reach his evil goal?