Discrete Mathematics **DSBA**

Seminar 13. Countable sets

1. Prove that the following sets are countable:

a) $\{n^2 \mid n \in \mathbb{N}\}$

b) $\{(x, y) \mid x \in \mathbb{Z},\ y \in \{1, 2, 3\}\}$

c) $\{(x, y) \mid x \in \mathbb{Z},\ y \in \mathbb{N}\}$

2. It is known that $A \subseteq B$ and $A \neq B$. Is it possible that A and B are of the same cardinality?

3. Prove that if A is an infinite set and B is a finite set, then the set $A \setminus B$ has the same cardinality as A.

4. Is it true that if B is a finite set then a set $A \cup B$ has the same cardinality as A (for arbitrary A)? Try $A = \mathbb{N}$ at first.

5. Let A be an infinite set and B be a countable set. Is it true, that the set $A \cup B$ has the same cardinality as A?

6. Prove that if A and A' are of the same cardinality and B and B' are of the same cardinality then $A \times B$ and $A' \times B'$ are of the same cardinality.

7. Prove that the following sets are countable:

a) $\mathbb{Z} \times \mathbb{Q}$

b) $\mathbb{N}^k = \underbrace{\mathbb{N} \times \mathbb{N} \cdots \mathbb{N}}_{k}$

c) $\{\{p, q\} \mid p, q \in \mathbb{Q}, p \neq q\}$

8. Prove that the set of finite subsets of rational numbers is countable.

9. Let A be a finite set and B be a countable set. Prove that the set of totally-defined functions $f \colon A \to B$ is countable.

10. Is it possible that B is not of the same cardinality that B', but the sets $A \times B$ and $A' \times B'$ are of the same cardinality (for some A)?

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Home Assignment 13

1. Are the following sets countable?

a) The set of prime numbers; **b)** $\{n \mid n \text{ is a prime or an even number }\}$.

2. Prove that $\mathbb{N} \times \mathbb{Z} \times \mathbb{Q}$ is a countable set.

3. Let $A \setminus B$ be an infinite set and B be a finite set. Does it imply that $A \setminus B$ and A are of the same cardinality (for arbitrary A and B)?

4. Let A be an infinite set and B be a countable set. Does it imply that $A \triangle B$ and A are of the same cardinality (for arbitrary A and B)?

5. Let A be an infinite set. Is it true, that for some countable subset $B \subseteq A$ there exists a surjection $f\colon A \setminus B \to A$?

6. Prove that any set of non-intersecting intervals ((a, b) and (c, d) have no common points) is either finite or countable.

7. Prove that any infinite set contains infinitely many non-intersecting countable subsets.

8. Let $f\colon A \to B$ be a totally-defined function. It is known that B is a countable set and for any $y \in B$ the preimage $f^{-1}(y)$ is either a finite or a countable set. Prove that A is either finite or a countable set.