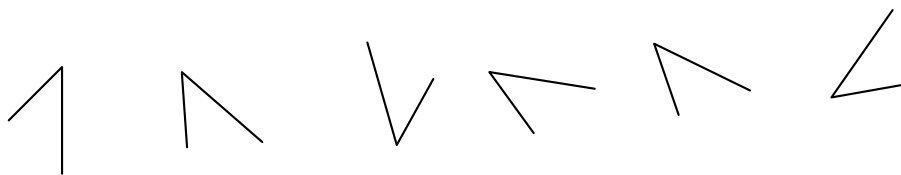


Seminar 14. Uncountable sets

1. Construct a bijection between a circle of radius 1 and a circle of radius $R > 0$.
2. Construct a bijection between the interval (a, b) and the set $(a, b) \cup \mathbb{Z}$.
3. Prove that \mathbb{R}^k and \mathbb{R} are of the same cardinality.
4. Is it true that the cardinality of the set of all lines of the plane is continuum?
5. Prove that the set of infinite zero-one sequences has the same cardinality as infinite sequences with elements from the set **a)** $\{0, 1, 2, 3\}$ **b)** $\{0, 1, 2\}$ **c)** \mathbb{N} .
6. Some one (may be with unlimited time and powers) has drawn curves of shape 8 on a plane without intersections. Prove that the set of drawn curves is countable.
7. Prove that the set of infinite sequences of real numbers has the same cardinality as \mathbb{R} .
8. Are the following sets of sequences $b_0, b_1, \dots, b_n, \dots$ countable?
 - a) zero-one sequences such that each continues subsequence of even length $b_i, b_{i+1}, \dots, b_{i+2k-1}$ contains equal number of zeroes and ones
 - b) zero-one sequences such that each continues subsequence of odd length $b_i, b_{i+1}, \dots, b_{i+2k}$ has almost equal number of zeroes and ones (difference between the greatest and the smallest number is at most one)?
9. Construct a bijection between a disc with border and disc without border.

Home Assignment 14

1. Let A be a point on a plane. Is it true that the set of (all) circles with center in A has cardinality continuum?
2. Prove that the set of intervals (a, b) on a line has cardinality continuum.
3. Let A be a set of (all) infinite sequences $a_0, a_1, \dots, a_n, \dots$ with elements 0, 1 or 2, such that $a_i \neq a_{i+1}$ for all i . Is it true, that the cardinality of A is continuum?
4. Prove that the cardinality of the set of bijections $\mathbb{N} \rightarrow \mathbb{N}$ is continuum.
5. Some one (may be with unlimited time and powers) has drawn on a plane figures of shape 1 (like on the picture) that do not intersect. Could the set of drawn figures have cardinality continuum?



6. A cross is a figure consisting of diagonals of a square (see the picture). Some one (may be with unlimited time and powers) has drawn on a plane crosses that do not intersect. Could the set of drawn crosses have cardinality continuum?

