Discrete Mathematics **DSBA**

Seminar 16. Probability II. Conditional Probability

Reminder. You shall choose a probability space before computing probability.

1. Four people A, B, C, D queue up in a random order (all the variants have the same probability). Find the conditional probability of that

a) A is the first given that B is the last

b) A is the first given that A is not the last

c) A is the first given that B is not the last

d) A is the first given that B stands in the queue later than A

e) A stands earlier than B given C stands in the queue later than A

2. Give examples of probability spaces and events A and B such that

a) $\Pr[A \mid B] > \Pr[A]$

b) $\Pr[A \mid B] < \Pr[A]$

c) $\Pr[A \mid B] = \Pr[A]$.

3. A dice is thrown. Are the events «the outcome number is even» and «the outcome number is divisible by 3» independent?

4. A player comes to a casino and randomly chooses one of the three slot machines (all the variants have the same probability). The casino owner knows [1] that the player wins the jackpot with probability 1/3 if the first slot machine chosen, with probability 1/2 if the second slot machine chosen and the chance to hit the jackpot for the third machine depends on the result of the player who is now leaving: with equal probability he will leave the machine in the configuration from which the probability of winning the jackpot is 2/3, or in the configuration with probability of the jackpot 1/2. Find the player's probability to win the jackpot.

5. There are three identical bags. There are two gold coins in the first one, one gold and one silver coin in the second one, and two silver ones in the third one.

You chose randomly one of the bags and then randomly pick a coin from it. It is golden. What is the probability that the second coin in the selected bag is golden? (At each choose the distribution is uniform)

6. One chooses a random permutation $x_1, x_2, \ldots, x_{49}$ of numbers from 1 to 49 (uniform distribution). Are the following events independent?

a) «$x_{24} > x_{25}$» and «$x_{25} > x_{26}$»

b) «x_{24} is greater than all the rest elements $(x_25, \ldots, x_49)$ » and «x_{25} is greater than all the rest elements$>$ $(x_26, \ldots, x_49)>$

7. Let A and B be independent events and it is known that $\Pr[\overline{B}] > 0$. Prove that the events A and $\overline{B}$ are independent.

8. Let events A and B have positive probability that is less than 1. Prove that if you know any three probabilities from the list $\Pr[A \mid B]$, $\Pr[A \mid \bar{B}]$, $\Pr[B]$, $\Pr[B \mid A]$ you can compute the fourth probability (which is unknown).

[1] In some countries, in order to avoid casino fraud, the probabilistic model of the slot machine must be strictly specified. Therefore, the probability of slot machine's transition to a winning configuration is uniquely determined by the current configuration.

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Home Assignment 16

Reminder. You shall choose a probability space before computing probability.

1. Find the probability that a uniformly random number from 1 to 100 is devisable by 2 given that it is devisable by 3?

2. A lottery organized as follows. Five numbers are uniformly random chosen from the set $\{1, 2 \ldots, 36\}$. Are the events «2 is one of the chosen numbers» and «3 is one of the chosen numbers» independent?

3. A totally defined function $f\colon \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ is chosen randomly. Are the events «f is an injection» and «$f(1) = 1$» independent?

4. Content blocking system determines a malicious file with 99% probability correctly (two-way error: malicious file is blocked with 99% probability, and a benign one is blocked with 1% probability).

It is known that the proportion of malicious files is 10^{-3}. Find probabilities of events «blocked benign file» and «blocked malicious file» given that the file is blocked (that is, recognized as malicious).

5. A jury of three members must make one of two possible decisions, one of which is correct. Two members of the jury independently make the right decision with probability p, and the third randomly chooses one of two possible solutions with equal probabilities. The final decision of the jury is made by the majority of votes. What is the probability that the jury will make the right decision? (Compare it with the probability p of the correct decision made by only one conscientious member of the jury.)

6. There are two decks of cards. One is complete, but the other is missing the ace of spades. Suppose you pick one of the two decks with equal probability and then select a card from that deck uniformly at random. What is the probability that you picked the complete deck, given that you selected the eight of hearts?

7. Two players play a match consisting of 20 games; A player wins if he is the first who gets 10 points (one point is given for winning, zero for losing, no draw happens). Counting all the options (any combination of twenty wins and losses) equiprobable, find the probability that the first player wins the match if after 15 games the score was $8 : 7$ in his favor.