## Plan week 15: Probability I

## Plan

- 1. Probability space
  - Outcomes
  - Sample space U
  - Probability is a function  $P: U \to [0, 1]$
  - Uniform probability space (all outcomes have the same probability  $\frac{1}{|U|}$ )
- 2. Examples: probability spaces for tossing one coin, tossing 5 coins, throwing one dice, throwing two dices.
- 3. Events. Probability of an event.
- 4. Examples
  - Probability of getting at least 9 points as a result of throwing two dices
  - Probability of the event «The third coin's side is head» after tossing 5 coins
  - Probability of getting exactly 3 heads as a result of tossing five coins
  - Probability of getting at least 3 heads as a result of tossing five coins
- 5. Probability of union:  $P[A \cup B] = P[A] + P[B]$  if  $A \cap B = \emptyset$ .
- 6. Example: probability of getting at least one even number after throwing two dices
- 7. Inclusion-exclusion principle for probabilities. Application for upper and lower bounds.
- 8. Using symmetry in the case of uniform probability space
- 9. Trees and subsequent choices. Formal computation of probability of the event «The k-th coin's side is head after tossing n coins».
- 10. Permutations
  - Example: students are passing exam with the tickets. The probability of getting the first element for the first student is the same as for the last student.

## References

The books are listed on the wiki-page.

- [4]: Sections 10.1, 10.2
- **[8]:** Chapter 14
- **[7]**: Sections 7.1, 7.2
- [1]: Sections 5.1
- [2]: Sections 5.1, 5.2
- [3]: Section 8.5