Plan week 17: Probability III. Expectations of Random Variables **DSBA 2019**

Plan

1. Random variables
   - Definition a random variable is a function $f : U \to \mathbb{R}$
   - Example. Dice throwing.
   - Example. Five coins throwing f_{5c}.
2. Expectation.
   - Definition I: $\mathrm{E}[f] = \sum_{u \in U} f(u) \times \Pr(u)$
   - Definition II: $\mathrm{E}[f] = \sum_{y \in f(U)} y \times \Pr[\{u \mid f(u) = y\}]$
3. Example $\mathrm{E}[f_{5c}]$.
4. Linearity of Expectation
   - Proof of $\mathrm{E}[f + g] = \mathrm{E}[f] + \mathrm{E}[g]$
   - Computing $\mathrm{E}[f_{5c}]$ via linearity: $f_5c(u) = g_1(u) + g_2(u) + \ldots + g_5(u)$ where $g_i((x_1, x_2, \ldots, x_5)) = x_i$. So $\mathrm{E}[f_{5c}] = 5\mathrm{E}[g_i] = \frac{5}{2}$.
   - Indicator variable: $I_A(u) = 1$ if $u \in A$ and $I_A(u) = 0$ if $u \notin A$. $\mathrm{E}[I_A] = \Pr[A]$.
   - Birthday paradox
5. Lemma: $\min(f) \leqslant \mathrm{E}[f] \leqslant \max(f)$
6. Markov inequality: $\Pr[f \geqslant \alpha] \leqslant \frac{\mathrm{E}[f]}{\alpha}$ for a non-negative random variable f.

References

The books are listed on the wiki-page.

[4]: Section 10.5

[8]: Chapter 19 and Section 20.1

[7]: Section 7.4

[2]: Sections 5.4