Plan and keywords week 5 **DSBA 2018**

1. The number of paths on coordinate plane from $(0,0)$ to (i,j) by moves up and right is $\binom{i+j}{i}$.
2. Pascal triangle and it's properties
   - symmetry
   - binomial coefficients increase from the left to center (and from the right to center)
   - lower bound for the central coefficient: $\binom{2n}{n} > \frac{2^{2n}}{n+1}$.
3. Binomial theorem and binomial coefficients
   - recursive formula
   $$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \qquad (1)$$
   - sum of binomial coefficients (in one row) and its combinatorial meaning
   - sign-alternating sum[1] of binomial coefficients
4. Combinatorial proofs
   - Prove of the recursive formula (1) via paths on the coordinate plane
   - a problem about a group of students with the leader: $n \times 2^{n-1} = \sum_{k=1}^{n} k\binom{n}{k}$;
   - $\sum_{k=1}^{n} \binom{n}{k}^2 = \binom{2n}{n}$;
5. Fibonacci numbers. Example of cuts of a celled strip $2 \times n$

References

The books are listed on the wiki-page.

[1]: Chapter 3 (except 3.4 that would be on the next lecture)

[4]: Sections 2.7, 2.8 (except 2.8.7 that would be on the next lecture) and 2.9 (only the beginning)

Keywords

- Binomial coefficients
- Pascal triangle
- Binomial theorem
- Combinatorial proofs
- Permutations
- Recursive formula (recurrence relation)
- Fibonacci number

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[1] adding and subtracting binomial coefficients alternatingly