Seminar 12. Zero Variant of the Winter Exam

Preambula. Recall that answers without arguments are not considered as solutions. You shall use convincing arguments so, that your solution becomes close to a mathematical proof.

1. Count the number of distinct undirected graphs on 10 vertices $v_1, \ldots, v_{10}$ that have (exactly) 4 edges and there is a vertex that is adjacent to each edge.

2. It is known that A, B, C are finite sets and $f\colon A \to B$ and $g\colon B \to C$ are surjections. It is known that $g(f(x))$ is a constant function. Find the cardinality of the set C.

3. A tournament is a directed graph such that each pair of edges is connected by exactly one edge (for each $u, v \in V$ either $(u, v) \in E$ or $(v, u) \in E$). Find the maximal number of vertices of degree 0 that a tournament on 2018 vertices can have?

4. It is known for sets A, B and C that a symmetric difference of each pair of sets contains the third set. Is it true that at least two of the sets do not intersect?

5. Do there exist a set A and a binary relation $R \subseteq A \times A$ such that the relation $R \circ R$ is transitive, but R is not transitive.

6. In how many ways can you take 5 numbers from the set $\{1, 2, \ldots, 36\}$ so that the difference between each pair of them is at least 8? (We subtract the smallest integer from the greatest one).

7. Prove that the inequality $k^k(n-k)^{(n-k)}\binom{n}{k} \leqslant n^n$ hold for arbitrary $n > k \geqslant 1$.

Hint. You can find a combinatorial proof that uses words of length n over the n-ary alphabet.

8. Find the number (of may be not totally defined) function f from $\{1, \ldots, 7\}$ to $\{1, \ldots, 7\}$ such that $f(\{1, 2, 3\}) = \{4, 5, 6\}$ and $f^{-1}(\{1, 2, 3\}) = \{4, 5, 6\}$ (there are no extra conditions on $f(7)$ and $f^{-1}(7)$).